Roll Number		

SET C

INDIAN SCHOOL MUSCAT
FIRST PRELIMINARY EXAMINATION
MATHEMATICS

CLASS: XII Sub. Code: 041 Time Allotted: 3 Hrs

10.01.2019 Max. Marks: 100

General Instructions:
(i) All questions are compulsory.
(ii) Questions in section A are very short answer type questions carrying 1 mark each.
(iii) Questions in section B are short- answer type questions carrying 2 marks each.
(iv) Questions in section C are long answer I type questions carrying 4 marks each.
(v) Questions in section D are long answer II type questions carrying 6 marks each.

SECTION- A (Questions 1 to 4 carry 1 mark each)

1. The equation of a line is given by $\frac{4-x}{2} = \frac{y+3}{3} = \frac{z+2}{6}$. Find the direction cosines of a line parallel to the above line.

2. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

3. A coin is tossed three times. Find $P(E|F)$ if, E = at least two heads; F = at most two heads.

OR

A card is drawn from a well- shuffled deck of 52 cards. If event E is that the card drawn is a spade and event F is that the card drawn is an ace. Show that the two events are independent.

4. If $\frac{1}{2}\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1} x$, find the value of x.

SECTION- B (Questions 5 to 12 carry 2 marks each)

5. Find the slopes of the tangent and the normal to the curve $y = \frac{(x-2)(x+1)}{(x+3)}$ at the points where it cuts the X-axis.

OR

Prove that the function $y = \left[\frac{4\,sinx}{2+\cos x} - x\right]$ is an increasing function of x in $\left(0, \frac{\pi}{2}\right)$.

6. Find the value of λ, so that the following two lines are perpendicular:
$\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1}$ and $\frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}$.

7. Form the differential equation of the family of circles touching the Y- axis at origin.

8. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.

9. Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$

10. Show that the vectors $\vec{a}=-2\hat{\imath}-2\hat{\jmath}+4\hat{k}$, $\vec{b}=-2\hat{\imath}+4\hat{\jmath}-2\hat{k}$ and $\vec{c}=4\hat{\imath}-2\hat{\jmath}-2\hat{k}$ are coplanar.

11. Find the equation of the plane that contains the point (-1, 3, 2) and perpendicular to each of the planes $x+2y-3z=5$ and $3x+3y-z=0$.

OR

Find the equation of the plane passing through the intersection of the planes $3x-y+2z-4=0$ and $x+y+z-2=0$ and through the point (2, 2, 1).

12. Find the mean and variance of the number of heads in three tosses of a fair coin.

SECTION- C (Questions 13 to 23 carry 4 marks each)

13. Show that: $tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]=\frac{\pi}{4}+\frac{1}{2}cos^{-1}x^2$

14. If $\vec{a}=\hat{\imath}+4\hat{\jmath}+2\hat{k}$, $\vec{b}=3\hat{\imath}-2\hat{\jmath}+7\hat{k}$ and $\vec{c}=2\hat{\imath}-\hat{\jmath}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d}=18$.

15. Using properties of determinants, prove that:

$$\begin{vmatrix}a+bx^2 & c+dx^2 & p+qx^2\\ ax^2+b & cx^2+d & px^2+q\\ u & v & w\end{vmatrix}=(x^4-1)\begin{vmatrix}b & d & q\\ a & c & p\\ u & v & w\end{vmatrix}$$

OR

Using properties of determinants, solve for x: $\begin{vmatrix}x-2 & 2x-3 & 3x-4\\ x-4 & 2x-9 & 3x-16\\ x-8 & 2x-27 & 3x-64\end{vmatrix}=0.$

16. Find the particular solution of the differential equation given that $y=1$ when $x=1$:
$(3xy+y^2)dx+(x^2+xy)dy=0.$

17. Evaluate : $\int\frac{x^2+5}{(x-1)(x+2)^2}dx$

18. If $f(x) = \begin{cases} ax + b & if \quad x > 2 \\ 7x - 4 & if \quad x = 2 \\ 3ax - 2b & if \quad x < 2 \end{cases}$ is continuous at $x = 2$, find the values of a and b.

19. Solve the differential equation: $cos^2 x \left(\frac{dy}{dx}\right) + y = tanx$.

OR

Solve the differential equation: $ye^y\, dx = (y^3 + 2xe^y)\, dy$, given that $y(0) = 1$.

20. Evaluate $\int_1^3 (3x^2 + 1)dx$ by the method of limit of sum.

21. In a group of 400 people, 160 are smokers and non-vegetarians, 100 are smokers and vegetarians and the remaining are non- smokers and vegetarians. The probabilities of getting a particular chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disease. What is the probability that the selected person is a smoker and non- vegetarian?

22. Differentiate $y = x^{tanx} + (tanx)^x$ and find $\frac{dy}{dx}$.

23. Evaluate: $\int_0^{\pi/2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

OR

Evaluate using properties of definite integrals : $\int_0^{\pi} \frac{x}{a^2 cos^2 x + b^2 sin^2 x} dx$

SECTION- D (Questions 24 to 29 carry 6 marks each)

24. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also find the point of intersection.

25. A binary operation $*$ is defined on the set X = R-{-1} by $x * y = x + y + xy,\ x, y \in$ X. Check whether $*$ is commutative and associative. Find the identity element and also find the inverse of each element of X.

OR

Let A = Q ×Q where Q is the set of rational numbers and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Then find:
(i) the identity element of $*$ in A.
(ii) invertible element of (a, b) and hence write the inverse of elements (5, 3) and $\left(\frac{1}{2}, 4\right)$.

26. Show that the semi- vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}(\sqrt{2})$.

27. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, find A^{-1} and use A^{-1} to solve the following system of equations:

$$x + 2y - 3z = -4,\ 2x + 3y + 2z = 2,\ 3x - 3y - 4z = 11.$$

OR

Using elementary transformation, find the inverse of the matrix: $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

28. A manufacturing company makes two types of teaching aids A and B of Mathematics for Class XII. Each type of aid A requires 9 hours for fabricating and 1 hour for finishing. Each type of aid B requires 12 hours for fabricating and 3 hours for finishing. For fabricating and finishing, the maximum hours available per week are 180 and 30 respectively. The company makes a profit of Rs.80 on each piece of type A and Rs.120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week?

29. Using integration, find the area of the region bounded by the line $2y = 3x + 12$ and the curve $3x^2 = 4y$.

OR

Using integration, find the area of the following region: $\left\{(x, y): |x - 1| \leq y \leq \sqrt{5 - x^2}\right\}$

End of the Question Paper